R&D NOTES

Temperature Dependency of Dynamic Coefficient for Nonequilibrium Capillary Pressure-Saturation Relationship

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Introduction

Formulation, evaluation, and correlation of the dynamic coefficient of two-phase fluid systems with temperature are provided. The dynamic coefficient is expressed as a function of the mobile wetting-phase saturation with parameters depending on temperature and validated for the system of perchloroethylene (PCE)/water flowing through coarse and fine sands.

Dependence of dynamic coefficient on temperature is needed for subsurface multiphase flow processes that undergo a nonequilibrium fluid distribution in porous media affected by temperature variation, for example, when fluids are injected at temperatures different than that of porous media leading to nonisothermal conditions (Civan[1]). Accurate correlation of data is instrumental and important because correlation can allow for prediction also at conditions other than those measured, help refining the values of measurements by virtue of averaging, although in an uncertain amount, and provide a means of evaluating the quality of data (Civan[2–7]).

This article provides an effective and practical method for correlation of the simulated data of Hanspal and Das[8] involving the temperature dependency of the flow of perchloroethylene/water through the coarse and fine sands. This is achieved by means of a proper formulation of the dynamic coefficient of two-phase fluid systems as a function of the mobile wetting-phase saturation with parameters depending on temperature. The correlation equation is derived by representation of relative permeability according to Brooks and Corey[9] which provides a satisfactory match of available data.

Formulation

The instantaneous change in fluid saturation (volume fraction of pore space occupied by the fluid) with time until attaining equilibrium in porous media occurs directly proportional to the difference between the dynamic and equilibrium capillary pressures and inversely proportional to the effective viscosity of the two-phase system of wetting and nonwetting fluids. Therefore, the rate equation expressing the variation of the wetting fluid saturation is given by

$$-\frac{dS_w}{dt} = k\left(p_c^{dyn} - p_c^{eqn}\right)\left(\frac{1}{\mu_{eff}}\right) \tag{1}$$

where S_w is the wetting fluid saturation, t is time, k denotes a rate coefficient (dimensionless), p_c^{dyn} and p_c^{eqn} denote the dynamic and equilibrium state capillary pressures, respectively, and μ_{eff} is the effective viscosity of the two-phase fluid system. The nonwetting fluid saturation is $S_{nw} = 1 - S_w$. Note that Eq. 1 conforms to the equation of Hassanizadeh and Gray[10] when rearranged as

$$p_c^{dyn} - p_c^{eqn} = -\tau\frac{dS_w}{dt} \tag{2}$$

where a group parameter τ in the units of viscosity, referred to as the dynamic coefficient, is defined by

$$\tau = \frac{\mu_{eff}}{k} \tag{3}$$

The mobility of the wetting and nonwetting fluids denoted by λ_w and λ_{nw}, and the effective mobility of the two-phase system denoted by λ_{eff} are defined, respectively, as

$$\lambda_w = \frac{Kk_{rw}}{\mu_w},\ \lambda_{nw} = \frac{Kk_{rnw}}{\mu_{nw}},\ \text{and}\ \lambda_{eff} = \frac{K}{\mu_{eff}} \tag{4}$$

where K is the absolute permeability of porous media, k_{rw} and k_{rnw} are the relative permeability, and μ_w and μ_{nw} are the absolute viscosity of the wetting and nonwetting fluid phases, respectively, and μ_{eff} denotes the effective viscosity of the two-phase fluid system. Then, the effective mobility of the two-phase system is given by a harmonic average mobility of the wetting and nonwetting fluids, as

$$\frac{2}{\lambda_{eff}} = \frac{1}{\lambda_w} + \frac{1}{\lambda_{nw}} \tag{5}$$

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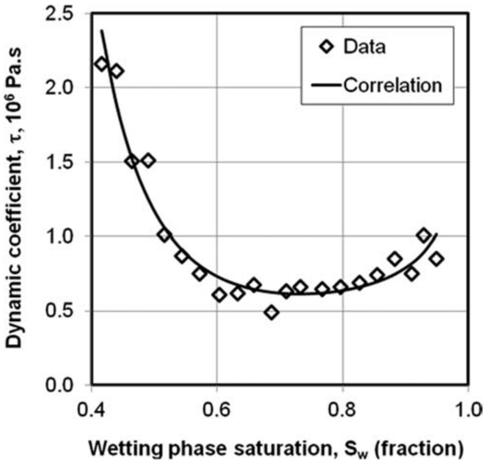

Figure 1. Correlation of the dynamic coefficient data of Goel and O'Carroll[11] for the system of 0.65 cSt oil/water.

Finally, Eq. 5 can be expressed as the following by invoking Eq. 4

$$\mu_{eff} = \frac{\mu_w}{2k_{rw}} + \frac{\mu_{nw}}{2k_{rnw}} \tag{6}$$

Brooks and Corey[9] describe the dependence of the relative permeability on the normalized mobile fluid saturation by the following empirical power-law equations

$$k_{rw} = k^o_{rw}(S_{Dw})^n \text{ and } k_{rnw} = k^o_{rnw}(S_{Dnw})^m = k^o_{rnw}(1 - S_{Dw})^m \tag{7}$$

where k^o_{rw} and k^o_{rnw} are empirical coefficients, and n and m are empirical exponents, and the normalized mobile wetting and nonwetting fluid saturations are given, respectively, by

$$S_{Dw} = \frac{S_w - S_{wi}}{1 - S_{nwi} - S_{wi}} \text{ and } S_{Dnw} = 1 - S_{Dw} \tag{8}$$

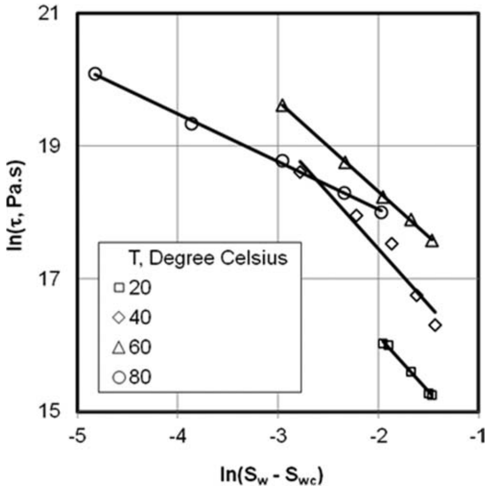

Figure 2. Correlation of the dynamic coefficient of coarse sand using a power-law function of the mobile fluid saturation with the coefficients of regressions and correlations given in Table 1.

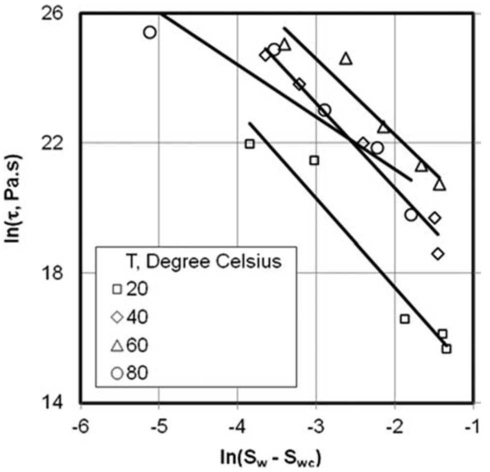

Figure 3. Correlation of the dynamic coefficient of fine sand using a power-law function of the mobile fluid saturation with the coefficients of regressions and correlations given in Table 1.

where S_{wi} and S_{nwi} denote the irreducible or immobile wetting and nonwetting fluid phase saturations, respectively. Substituting Eq. 7 into Eq. 6 yields

$$\mu_{eff} = \frac{\mu_w}{2k^o_{rw}(S_{Dw})^n} + \frac{\mu_{nw}}{2k^o_{rnw}(1 - S_{Dw})^m} \tag{9}$$

Then, invoking Eq. 9 into Eq. 3 yields

$$\tau = \frac{\tau_w}{(S_{Dw})^n} + \frac{\tau_{nw}}{(1 - S_{Dw})^m}, n > 0, m > 0 \tag{10}$$

where the new group parameters τ_w and τ_{nw} are defined by

$$\tau_w = \frac{\mu_w}{2k^o_{rw}k}, \tau_{nw} = \frac{\mu_{nw}}{2k^o_{rnw}k} \tag{11}$$

Data Correlation Method, Discussion, and Conclusions

The validity of Eq. 10 is proven by satisfactorily correlating the dynamic coefficient data of Goel and O'Carroll[11] for the system of 0.65 cSt oil/water using $\tau_w = 0.021$ (10^6 Pa.s), $\tau_{nw} = 0.3$ (10^6 Pa.s), $n = 5.2$, and $m = 0.4$ as shown in Figure 1. The average values of data over the bounds of

Table 1. Coefficients of Regressions and Correlations of the Power-Law Parameters given in Figures 2 and 3 at Various Temperatures

T°C	a	b	S_{wc}	R^2
Coarse sand				
20	1.677	12.780	0.050	0.993
40	1.682	14.094	0.155	0.949
60	1.363	15.586	0.200	1.000
80	0.720	16.604	0.325	0.998
Fine sand				
20	2.734	12.084	0.050	0.947
40	2.605	15.401	0.130	0.976
60	2.345	17.548	0.175	0.925
80	1.613	17.967	0.290	0.835

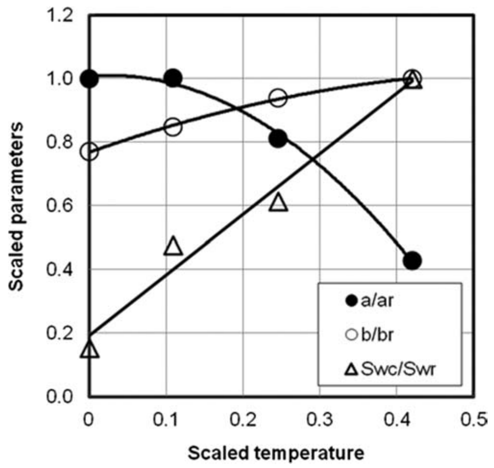

Figure 4. Correlation of the temperature dependency of the power-law parameters of coarse sand with the coefficients of regressions and correlations given in Table 2.

confidence intervals as reported by Goel and O'Carroll[11] were used here.

Hanspal and Das[8] numerically investigated the flow of perchloroethylene/water system through coarse and fine sands over the temperature range of 20°C to 80°C. This provides simulated data for the dynamic coefficient of the nonequilibrium capillary pressure vs. saturation relationship. An examination of the data of Hanspal and Das[8] reveals that only the first term is dominant in Eq. 10 for their data and, therefore, the dynamic coefficient τ (Pa.s) can be expressed by an empirical inverse power-law function of the mobile wetting-phase saturation as, by neglecting the second term on the right of Eq. 10 and then substituting Eq. 8 and $n \equiv a$

$$\tau = \frac{\tau_o}{(S_w - S_{wi})^a}, a > 0 \qquad (12)$$

where the new group coefficient τ_o is defined by

$$\tau_o = \tau_w (1 - S_{nwi} - S_{wi})^a, a > 0 \qquad (13)$$

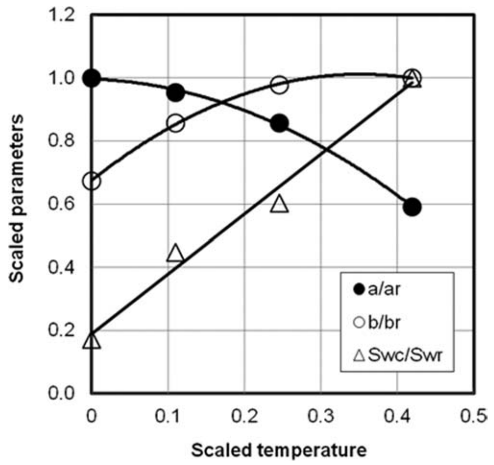

Figure 5. Correlation of the temperature dependency of the power-law parameters of fine sand with the coefficients of regressions and correlations given in Table 2.

Table 2. Coefficients of Regressions and Correlations of the Temperature Dependency of the Power-Law Parameters given in Figures 4 and 5

Parameters	Coarse Sand	Fine Sand
T_r	20	20
T_c	223	223
a_1	−3.763	−2.114
a_2	0.191	−0.0756
a_3	1.008	0.996
a_r	−1.677	−2.734
R^2	0.997	0.999
b_1	−0.726	−2.73
b_2	0.86	1.917
b_3	0.768	0.675
b_r	16.60	17.97
R^2	0.999	0.999
c_1	1.911	1.893
c_2	0.192	0.190
S_{wr}	0.325	0.290
R^2	0.975	0.984

The empirical fitting parameters of Eq. 12, namely a, $b \equiv \ln \tau_o$, and $S_{wc} \equiv S_{wi}$, are dependent on temperature. Figures 2 and 3 show the correlation of their simulated data for the dynamic coefficients of the coarse and fine sands, respectively, at various temperatures using Eq. 12. Table 1 presents

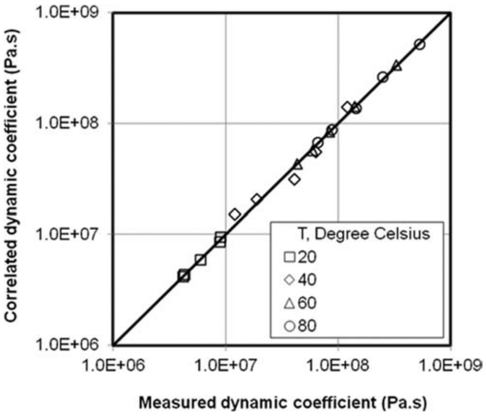

Figure 6. Comparison on the logarithmic scale of the correlated vs. measured dynamic coefficients of the coarse sand reveals a good agreement.

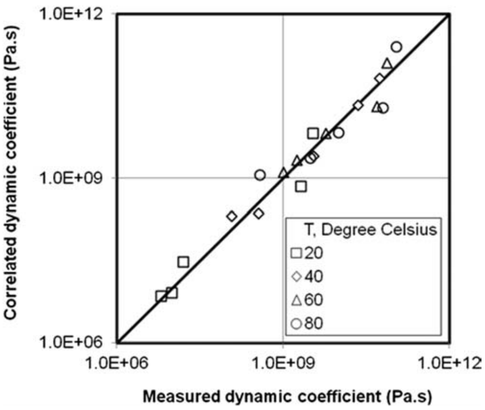

Figure 7. Comparison on the logarithmic scale of the correlated vs. measured dynamic coefficients of fine sand.

the coefficients of regressions and correlations of the power-law parameters. Figures 4 and 5 show the correlations of the temperature dependency of the power-law parameters of the coarse and fine sands, respectively, with the correlations and coefficients of regressions given in Table 2. As can be seen, the temperature dependence of the scaled parameters can be expressed by the following correlations

$$\frac{a}{a_r} = a_1 x^2 + a_2 x + a_3 \tag{14}$$

$$\frac{b}{b_r} = b_1 x^2 + b_2 x + b_3 \tag{15}$$

$$\frac{S_{wc}}{S_{wr}} = c_1 x + c_2 \tag{16}$$

where x is the scaled temperature given by

$$x = \frac{T_r - T_c}{T - T_c} - 1 \tag{17}$$

Here, T_c and T_r denote the asymptote and reference temperatures, respectively, determined by best regression of data; and a_r, b_r, and S_{wr} are the maximum values of a, b, and S_w, respectively; and a_1, a_2, a_3, b_1, b_2, b_3, c_1, and c_2 are empirical fitting constants. Figures 6 and 7 show that the correlated vs. measured dynamic coefficients are in a good agreement. This proves the validity of this formulation considered for correlation of the dynamic coefficient as a function of fluid saturation and temperature.

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